Name:______________________________

Vector/ Newton's Law Packet

- Creating a Vector
  - Questions #1-4 ____
- Finding a Vector's Component
  - Questions #5-8____
- Finding a Vector's Magnitude and Direction
  - Questions #9-12____

Chapter 3

Vectors: Knowing Where You're Headed

In This Chapter

- Understanding what makes a vector
- Expressing vectors in different forms
- Converting vectors to different forms
- Adding vectors
- Expressing motion as vectors

Although an object in motion has a speed, it also has a direction. Together, the speed and the direction describe the object's motion. For example, you may be heading off to your grandmother's house at 60 miles/hour, but unless you're pointed in the right direction, you're not going to get there.

Physics takes note of the fact that objects in motion need two quantities to fully describe that motion — speed and direction — by saying that such motion is a *vector* quantity.

Creating a Vector

A *vector* is a combination of exactly two values: a magnitude (like the speed of an object in motion) and a direction (such as the direction of an object in motion). All kinds of things can be described with vectors, including constant motion, acceleration, displacement, magnetic fields, electric fields, and many more.

Vectors are defined by a *magnitude* (the length of the vector) and a direction. For example, take a look at the vector in Figure 3-1.

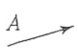

Figure 3-1: A vector.

In physics, vectors are written in bold type. The vector in Figure 3-1 — I'll call it **A** — represents the displacement of a golf ball from the tee. Its length is 100 yards, and its direction is 15° north of due east. That's all you need to have a vector — a magnitude and a direction.

Now take a look at the two vectors in Figure 3-2, **A** and **B**. These two vectors are considered equal, which is written as **A** = **B**.

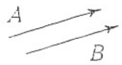

Figure 3-2: Two vectors.

REMEMBER

Two vectors are considered equal if they have the same magnitude and direction. They do *not* need to start at the same point. The magnitude of a vector **A** — that is, its length — is written as A, not in bold type.

Figure 3-3 shows the standard coordinate system for vectors. Note the x and y axes, which vectors are measured against.

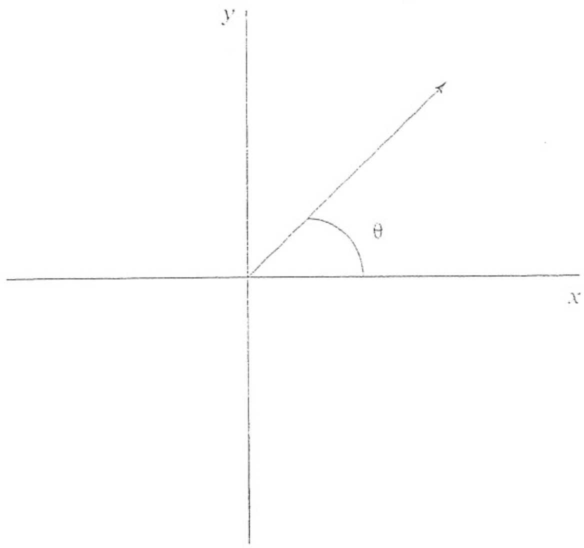

Figure 3-3: Vector coordinate system.

The x and y axes are measured using some standard physics units, such as centimeters. Positive x (also called *east*) is to the right, negative x is to the left; positive y (also called *north*) is up, negative y is down. The center of the graph, where the axes meet, is called the *origin*. A vector is commonly described by its length and its angle from the positive x axis (0° to 360°).

EXAMPLE

Q. Suppose the vector in Figure 3-3 is 3.0 centimeters long and at an angle of 45° with respect to the x axis. How would you exactly describe this vector?

A. The correct answer is 3.0 centimeters long and at an angle of 45° with respect to the x axis.

1. A marble starts at the origin and rolls 45 meters east. Describe where it ends up, in vector notation.

2. A marble starts at the origin and rolls 45 meters east. Then it moves 90 meters west. Describe where it ends up, in vector notation.

Understanding Vector Components

In addition to specifying a vector with a magnitude and a direction, you can specify it with a pair of coordinates as measured from the origin. For example, take a look at the vector in Figure 3-4, where the measurements are in centimeters.

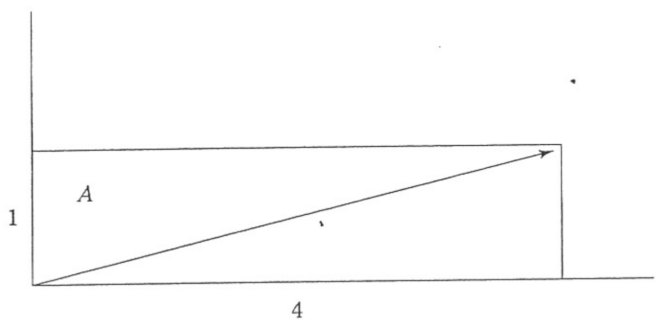

Figure 3-4: Resolving a vector.

You can describe the vector in Figure 3-4 with a length and an angle, of course, but you also can describe it by the coordinates of the tip of its arrow. In this case, that tip is at 4 centimeters to the right and 1 centimeter up from the origin. You notate that location as (4, 1), which is a valid way of expressing a vector.

So the two ways of expressing a vector are

- As a magnitude and a direction; for example, **A** is a vector along the x axis of length 5
- As a pair of coordinates corresponding to the tip of the vector (assuming the tail of the vector is at the origin); for example, **A** = (5, 0)

EXAMPLE

Q. Suppose a person walks 3.0 meters to the right of the origin. What is his displacement vector in terms of coordinates?

A. The correct answer is (3.0, 0).

The person's x coordinate is 3.0, and his y coordinate is 0, so his vector displacement is (3.0, 0).

---

3. A marble starts at the origin and moves to the right 5.0 centimeters. What is its new displacement, in vector coordinate terms?

4. Suppose you move to the right of the origin by 3.5 meters and then up 5.6 meters. What is your final vector from the origin, in coordinate terms?

Finding a Vector's Components

You can convert from the magnitude/angle way of specifying a vector to the coordinate way of expression. Doing so is essential for the kinds of operations you can expect to execute on vectors, such as when adding vectors together.

For example, you have one vector at 15° and one at 19°, and you want to add them together. How the heck do you do that? If you were to convert them into their coordinates, (a, b) and (c, d), the answer would be trivial because you only have to add the x and y coordinates to get the answer: (a + c, b + d).

To see how to convert between the two ways of looking at vectors, take a look at vector **v** in Figure 3-5. The vector can be described as having a magnitude v at an angle of θ.

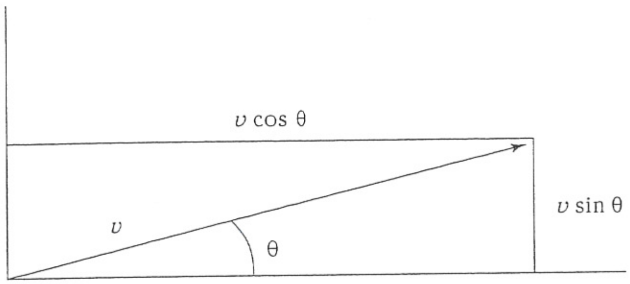

Figure 3-5: Finding a vector's components.

To convert this vector into the coordinate way of looking at vectors, you have to use the trigonometry shown in the figure. The x coordinate equals v cos θ, and the y coordinate equals v sin θ:

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

Keep this relationship in mind because you'll come across it often in physics questions.

Q. Suppose that you've walked away from the origin so that you're now at 5.0 kilometers from the origin, at an angle of 45°. Resolve that into vector coordinates.

A. The correct answer is (3.5, 3.5).

1. Apply the equation $v_x = v\cos\theta$ to find the x coordinate. That's 5.0·cos 45°, or 3.5.
2. Apply the equation $v_y = v\sin\theta$ to find the y coordinate. That's 5.0·sin 45°, or 3.5.

5. Resolve a vector 3.0 meters long at 15° into its components.

6. Resolve a vector 9.0 meters long at 35° into its components.

7. Resolve a vector 6.0 meters long at 125° into its components.

8. Resolve a vector 4.0 meters long at 255° into its components.

Finding a Vector's Magnitude and Direction

If you're given the coordinates of a vector, such as (3, 4), you can convert it easily to the magnitude/angle way of expressing vectors using trigonometry.

For example, take a look at the vector in Figure 3-5. Suppose that you're given the coordinates of the end of the vector and want to find its magnitude, v, and angle, θ. Because of your knowledge of trigonometry, you know that

$$x = v \cdot \cos\theta$$

$$y = v \cdot \sin\theta$$

In other words, you know that

$$\frac{x}{v} = \cos\theta$$

$$\frac{y}{v} = \sin\theta$$

Which means that

$$\theta = \sin^{-1}(y/v)$$

$$\theta = \cos^{-1}(x/v)$$

You can calculate the inverse sine ($\sin^{-1}$) or inverse cosine ($\cos^{-1}$) on your calculator. (Look for the $\sin^{-1}$ and $\cos^{-1}$ buttons.)

In Figure 3-5, you're given x and y, the coordinates, but not v, the magnitude. Dividing the expressions for y and x above gives you

$$\frac{y}{x} = \frac{\cancel{v}\sin\theta}{\cancel{v}\cos\theta} = \tan\theta$$

Where tan θ is the tangent of the angle. This means that

$$\theta = \tan^{-1}(y/x)$$

Suppose that the coordinates of the vector are (3, 4). You can find the angle θ as the $\tan^{-1}(4/3) = 53°$. And you can use the Pythagorean theorem to find the *hypotenuse* — the magnitude, v — of the triangle formed by x, y, and v:

$$v = \sqrt{x^2 + y^2}$$

Plug in the numbers for this example to get

$$v = \sqrt{3^2 + 4^2} = 5$$

So if you have a vector given by the coordinates (3, 4), its magnitude is 5, and its angle is 53°.

EXAMPLE

Q. Convert the vector given by the coordinates (1.0, 5.0) into magnitude/angle format.

A. The correct answer is magnitude 5.1, angle 78°.

1. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle. Plug in the numbers to get $\tan^{-1}(5.0/1.0) = 78°$.
2. Apply the Pythagorean theorem $v = \sqrt{x^2 + y^2}$ to find the magnitude. Plug in the numbers to get 5.1.

9. Convert the vector (5.0, 7.0) into magnitude/angle form.

10. Convert the vector (13.0, 13.0) into magnitude/angle form.

11. Convert the vector (–1.0, 1.0) into magnitude/angle form.

12. Convert the vector (–5.0, –7.0) into magnitude/angle form.

Adding Vectors Together

You're frequently asked to add vectors together when solving physics problems. To add two vectors, you place them head to tail and then find the length and magnitude of the result. The order in which you add the two vectors doesn't matter. For example, suppose that you're headed to the big physics convention and have been told that you go 20 miles due north and then 20 miles due east to get there. At what angle is the convention center from your present location, and how far away is it?

You can write these two vectors like this (where east is along the x axis):

$$(0, 20)$$

$$(20, 0)$$

In this case, you need to add these two vectors together, and you can do that just by adding their x and y components separately:

$$\begin{array}{r} (0,20) \\ +(20,0) \\ \hline (20,20) \end{array}$$

Do the math, and your resultant vector is (20, 20). You've just completed a vector addition. But the question asks for the vector in magnitude/angle terms, not coordinate terms. So what is the magnitude of the vector from you to the physics convention? You can see the situation in Figure 3-6, where you have v_x and v_y and want to find v.